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SIMPLIFIED CALCULATION OF BLACK-BODY RADIATION

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GENERAL ELECTRIC

SIMPLIFIED CALCULATION OF BLACK-BODY RADIATION

A radiation slide rule* has been compiled for direct computation of the spectral distribution and the amount of radiant energy associated with a desired temperature. Conversion scales for different energy and temperature units are also incorporated

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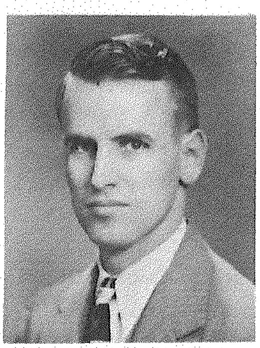

A. H. Canada

THE impetus given infrared development by the war has emphasized its usefulness in science and engineering. The need by engineers for standardization of terminology and of circuit techniques and characteristics of radiation-detection devices, so that they may be applied to infrared developments, has not only accented the need for an understanding and utilization of the Planck radiation functions for energy emitted by heated objects, but has also increased the necessity for simplification of these calculations.

In the application of bolometric and other types of sensitive radiation-detecting devices both to infrared spectroscopy and to military equipment there is occasion both for the expression of radiated energies, though they may be small, in equivalent terminology, and for rapid means of conversion and calculation. For example, if the minimum detectable signal is expressed in watts per square meter for one detector and Btu per square foot per hour for another device, one should not need to go through too many mathematical gymnastics to find a comparative basis between the two equipments. Furthermore, in the application of various types of photoemissive and photoconductive detectors to radiation measuring problems, the threshold spectral limit of sensitivity is particularly important.

This is also true in the case of the window, or lens, materials to be utilized in radiation measurement. If, for instance, it is desirable to detect the energy radiated by an object at a particular temperature and if one wishes to evaluate the lead sulfide cell (sensitive to about 3.5 microns) for the particular application, it is important to know the distribution of the energy in the spectrum.

RADIATION LAWS

The radiation slide rule (FIG. 1) described in this article is a simple device for computing these various energy relationships. It is based on the well-known Planck equation defining the spectral distribution of radiant energy, the Wien displacement law of shift of the radiation maximum with temperature, and the Stefan-Boltzmann law of total radiation.

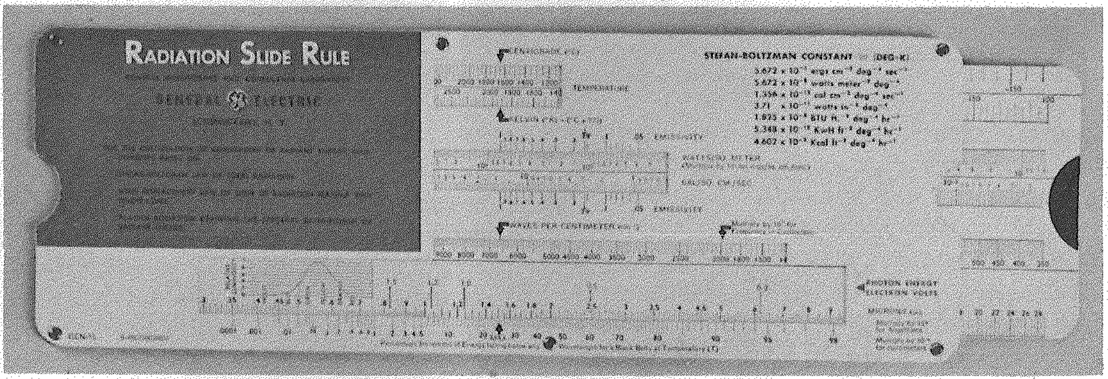

Fig. 1. The face of the radiation slide rule, containing energies and temperature in the metric system and the distribution-of-energy scale. The relative-luminosity curve appears at the left end of the micron scale

Planck's empirical equation

$$W_\lambda = C_1\lambda^{-5}\left(\epsilon^{\frac{C_2}{\lambda T}} - 1\right)^{-1} \qquad (1)$$

was derived to fit the distribution of energy in the spectrum from a black body at a particular tempera-

*The rule, designated GEN-15A, is printed on varnished card stock and is satisfactory for order-of-magnitude calculations. It may be obtained, postpaid, at $1.00 from the General Electric Company, 1 River Road, Schenectady, N. Y.

ture. Values and dimensions of the terms in this and later equations are given in the tabulation of nomenclature on the next page.

In lieu of a family of black-body radiation curves for various temperatures as classically illustrated, plotted with energy per unit area per unit wavelength as a function of wavelength, a ratio plot relative to λT is illustrated in FIG. 2. The data for this curve were obtained from *Miscellaneous Physical Tables—Planck's Radiation Functions*, sponsored by the National Bureau of Standards, where tables based on the λT relation are given. This curve is the spectral emittance ratio of energy per unit area per unit wavelength to energy per unit area per unit wavelength at the maximum point of the black-body distribution plotted against λT.

0.375. In the following ratio the temperature cancels out, since one temperature of radiator is considered in Equation (2):

$$\frac{\lambda T}{\lambda_m T} = \frac{(0.375)}{2897} \quad \begin{matrix} \leftarrow \text{Given argument.} \\ \leftarrow \text{Wien's law} \end{matrix} \tag{4}$$

By substituting this ratio, the value of $\lambda_m T$ and the given argument in Equation (2), the energy ratio can be obtained. When computations are based on the λT relation and the ratio in Equation (2), the first radiation constant cancels out so that state of polarization or size of solid angle does not enter.

By integration of the Planck Equation (1) with respect to wavelength from zero to infinity the total

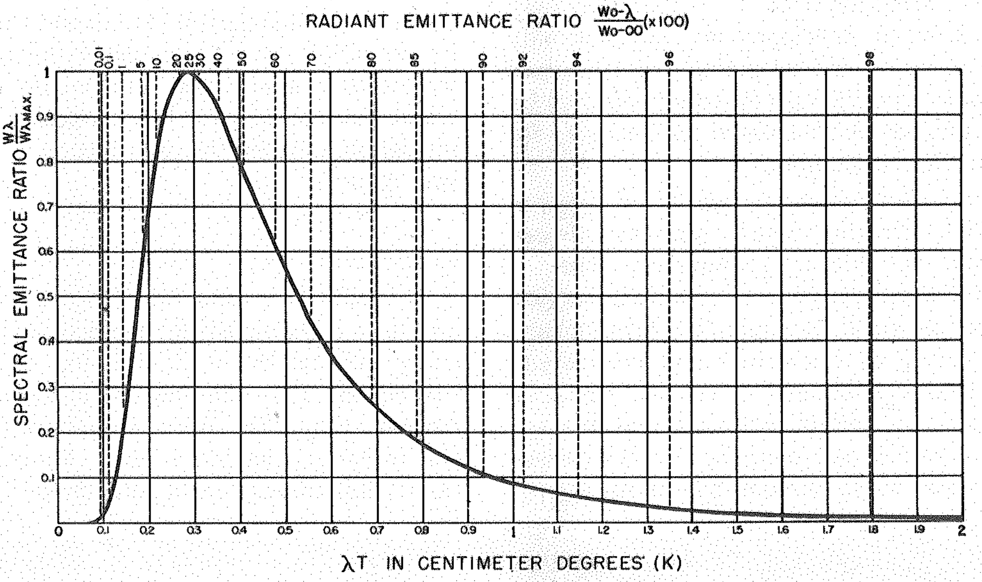

Fig. 2. Distribution of the energy radiated by a black body

Referring again to Planck's equation, the ratio is equal to

$$\frac{W_\lambda}{W_{\lambda m}} = \frac{C_1 \lambda^{-5} \left(\epsilon^{\frac{C_2}{\lambda T}} - 1 \right)^{-1}}{C_1 \lambda_m^{-5} \left(\epsilon^{\frac{C_2}{\lambda m T}} - 1 \right)^{-1}} \tag{2}$$

By setting the derivative of Equation (1) equal to zero and solving, the relationship of the wavelength to the temperature at which W_λ is maximum is obtained. This is Wien's displacement law and is expressed

$$\lambda_m T = 2897 \tag{3}$$

To compute the ratio in Equation (2) for a given argument of λT, consider the computation for λT at

radiated energy is obtained. This is then expressed in a lumped-constants relationship by the Stefan-Boltzmann law, which is

$$W = \rho \sigma T^4$$

or

$$W = \rho \sigma (T^4 - T_0^4) \tag{5}$$

This equation yields the total energy radiated to cold space, or to temperature T_0, from a unit surface into 2π solid angle. The energy is proportional to the emissivity factor (ρ), being the ratio of the energy from the given radiator to that from a black body at the same temperature. The various values of the Stefan-Boltzmann constant (σ), as used for a basis of the radiation slide rule, are given in Table I.

NOMENCLATURE

Constants applicable to the radiation laws according to R. T. Birge (*Rev. of Mod. Phys.*, **3**, 233, 1941)

W_λ = Radiant energy emitted per unit area per unit range of wavelength

W = Radiant energy emitted per unit area

C_1 = First radiation constant

$= 2\pi hc^2 = 3.740 \times 10^{-5}$ erg-cm^2 sec^{-1} when W_λ denotes the emission of unpolarized radiation in unit range of wavelength per unit surface in 2π solid angle

$= 8\pi hc = 4.990 \times 10^{-15}$ erg-cm when W_λ denotes energy density of unpolarized radiation

$= hc^2 = 5.950 \times 10^{-6}$ erg cm^{-2} sec^{-1} when W_λ denotes intensity of lineally polarized radiation in unit range of wavelength, per unit surface, per unit solid angle, perpendicularly to a surface

C_2 = 1.4385 cm-deg—second radiation constant

T = Absolute temperature of radiating body (°K)

T_0 = Absolute temperature of surroundings (°K)

λ = Wavelength in microns

λ_m = Wavelength in microns for maximum radiant energy

ϵ = 2.718—Napierian base

ρ = Emissivity factor (black body = 1)

σ = Stefan-Boltzmann constant

h = 6.624×10^{-27} erg-sec—Planck's constant of action

c = 2.99776×10^{10} cm sec^{-1}—velocity of light

TABLE I

STEFAN-BOLTZMANN CONSTANTS BASED ON DEGREES KELVIN FOR THE VARIOUS ENERGY SCALES ON THE RADIATION SLIDE RULE.

5.672×10^{-5} erg cm^{-2} deg^{-4} sec^{-1}
5.672×10^{-8} watts meter^{-2} deg^{-4}
1.356×10^{-12} cal cm^{-2} deg^{-4} sec^{-1}
3.71×10^{-11} watts in^{-2} deg^{-4}
1.825×10^{-8} Btu ft^{-2} deg^{-4} hr^{-1}
5.348×10^{-12} kw-hr ft^{-2} deg^{-4} hr^{-1}
4.602×10^{-9} kg-cal ft^{-2} deg^{-4} hr^{-1}

Restating these laws: Planck's equation defines the distribution of the energy in the spectrum or the shape of the black-body radiation curve; Wien's displacement law specifies the position of peak energy relative to wavelength; and the Stefan-Boltzmann law yields the total energy or area under the curve.

DESIGN OF RADIATION SLIDE RULE

To aid in the simplified handling of these radiation laws, the radiation slide rule has been compiled*. The principle of the device has been previously suggested and a few such slide rules constructed. As far as the author can determine, the idea was first proposed by M. Czerny, in Germany, and later rules were introduced in this country by G. Hass, of the Engineer Research and Development Laboratories at Fort Belvoir, Virginia. The slide rule described here has been materially expanded over those earlier rules to cover not only computation of total energy but the conversion of these values to other systems of units. Various temperature scales are included so that energy calculations can be based on scales other than Kelvin.

The Kelvin scale consists of two logarithmic cycles. A part of it is shown in FIG. 3. Aligned by the proper Stefan-Boltzmann constant are two total energy scales, watts per square meter and Btu per square foot per hour. These scales are also logarithmic. Since the total radiated energy is proportional to the fourth power of the absolute temperature, there are four log cycles for one log cycle of the Kelvin scale. These log cycles are positioned relative to the 1000 K point; thus, the total radiated energy for a body at 1000 K is 5.672×10^4 watts per square meter, or 1.825×10^4 Btu per square foot per hour. Other total energy scales, incorporated in the slide rule and available for ready conversion from one set of units to another, are those for which constants are given in Table I.

WAVELENGTH OF MAXIMA

Bearing a fixed relationship to temperature, as shown by Wien's displacement law, is the micron scale illustrated in FIG. 3. This, too, is a logarithmic cycle the same length as the logarithmic cycle of the Kelvin scale, but since wavelength maximum varies inversely with the absolute temperature, the logarithmic cycle is inverted.

For additional information, photon energies in electron volts are marked on the slide rule's micron scale to show their relationship to wavelength.

For use in spectroscopy, two of the scales make possible the conversion of microns to waves per centimeter. The waves-per-centimeter scale (reciprocal centimeters or wave numbers) consists of a logarithmic cycle inverted to the microns scale and of the same cycle length; thus, one micron is equivalent to 10,000 waves, 5 microns is equivalent to 2000 waves, etc.

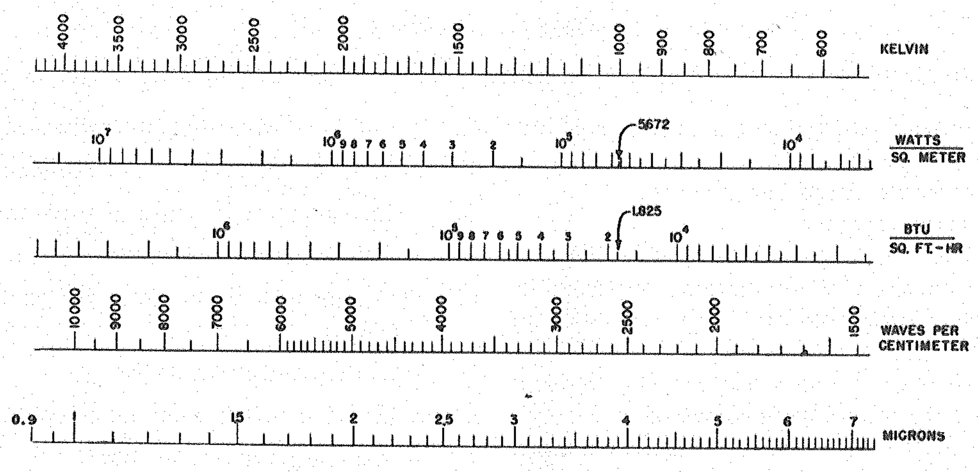

Fig. 3. Basic scales used on the radiation slide rule

Since the interrelationship of the various total energy scales is fixed, the radiation slide rule can be used for the conversion of energy units. To simplify the changing of temperature values, Centigrade, Fahrenheit, and Rankine scales are included in their proper correlation to the Kelvin scale and are usable for temperature conversion.

EMISSIVITY LESS THAN ONE

The radiation slide rule may also be used for computing total radiated energy other than that from a black body. This is illustrated in FIG. 4. If a given temperature for a radiating black body is set on the index—for example 2100 K, as in FIG. 4—the total radiated energy is read opposite the index as 1.1×10^6 watts per square meter. If, however, the radiating body at 2100 K is not a black body but radiates only 30 percent as well as a black body, its emissivity then is 0.3 and the total energy radiated is 3.2×10^5 watts per square meter.

These values of radiant energy are considered to be those radiated by a unit surface into a π solid angle or a hemispherical solid angle. To obtain the energy radiated per steradian, or unit solid angle, one must divide by π. Since the scales are logarithmic, this can be accomplished by multiplying by the reciprocal of π, which is marked on the emissivity scale.

SPECTRAL DISTRIBUTION OF ENERGY

Thus far the radiation slide rule is only a means of conversion, and, with the exception of the emissivity, this conversion could be accomplished in tabular form or as a nomograph of the several scales. The main reason for the sliding part of the rule is noted in FIG. 5. The micron scale is on the slider. On the fixed part of the rule is a scale for percentage increment of energy falling below any wavelength, which is used to determine the distribution of the energy within the spectrum for a black-body radiator.

The use of this scale may be explained by the example in FIG. 5. The scale is set for 1000 K. The number 2.89 on the micron scale is opposite the index, indicating that the maximum of the black-body energy distribution curve occurs at this wavelength. Twenty-five percent also occurs at the index, which indicates that 25

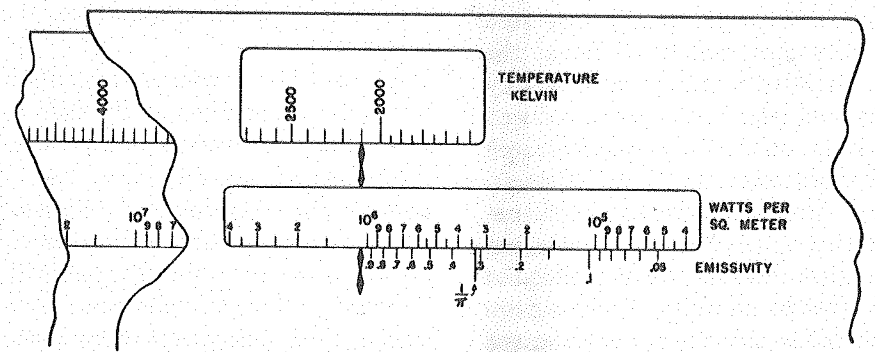

Fig. 4. A portion of the radiation slide rule illustrating the use of the emissivity scale and unit solid-angle mark

percent of the total radiated energy of the black body at 1000 K will fall on the short-wavelength side of this maximum at 2.89 while 75 percent, or the remaining energy, will fall on the long-wavelength side. This scale can be used to determine the amount of radiant energy falling between any two wavelengths. For example, the energy falling between 5.5 microns and 9.5 microns is 20 percent (90 minus 70) of the total radiated energy from this black body at 1000 K. At this setting, the slide rule shows that less than one ten-thousandth of one percent of the energy of the black body falls within the visible spectrum.

The distribution of the energy scale is applicable to any temperature setting of the slide rule. As higher temperatures are utilized, of course, a larger percentage increment falls within the visible portion of the spectrum.

The validity of this percentage-distribution scale depends upon the logarithmic construction of the slide rule. Illustrated in FIG. 2 is the radiant-emittance ratio of the energy between wavelength zero and λ to energy between zero and infinity. This is obtained by integration of Planck's equation within specific limits with respect to wavelength. The ratio may be shown to bear a fixed relationship to λT and is so plotted in FIG. 2. Since the micron scale is logarithmic, the spacings of the increment scale, corresponding with the spacings shown in FIG. 2, remain fixed for all settings of the temperature scale.

To increase the usefulness of the slide rule in the visible portion of the spectrum, a relative-luminosity curve for the eye is included on the visible-light portion of the micron scale (shown in FIG. 1). A separate nomogram printed on the rule shows specific characteristics of tungsten, giving color temperature and brightness in candles per square centimeter, as functions of the true temperature.

SAMPLE CALCULATIONS

Two problems may serve to illustrate the use of this radiation slide rule. The graphite plate of a particular metal-shell vacuum tube operates at 500 C. Carbon at 500 C has an emissivity of only about 0.08, or is only eight percent as good an energy radiator as a black body. If 500 C is set on the index, it is found that a black body radiates 2×10^4 watts per square meter while carbon, at its low value of emissivity, radiates 1.6×10^3 watts per square meter as read opposite the appropriate emissivity. This radiant energy is absorbed in the metal envelope of the tube. Assuming a temperature of 100 C at the metal, we may compute the energy radiated to the carbon by the metal. In this case, since the metal is the surrounding, the emissivity factor of the metal does not enter into the calculation. With 100 C set at the index the energy reading at the index is 1.1×10^3 watts per meter. By subtraction it is found that 500 watts per square meter is the net radiation retained at the metal envelope from plate dissipation.

If a glass envelope is used, the problem becomes much more complex, since some of the radiation is transmitted by the glass. It is then desirable to find how much of the radiated energy is transmitted. At 2.5 microns, the glass begins to be a poor transmitter of infrared radiation. On the percentage-increment scale, 2.5 microns for this particualr temperature (100 C) falls at 6 percent. Thus approximately 6 percent of the energy is transmitted by the glass, while 94 percent of the energy radiated by the plate is absorbed by the glass. Since the percentage-increment scale is for black-body distribution, the percentage of energy transmitted is only an approximation.

In another device it is desirable to use a lead-sulfide photoconductive detector to measure the radiant energy from a black-body radiator varying between 1000 and 2000 F. To determine the order of accuracy to be expected in such an application, the spectral characteristics of the lead-sulfide cell are important. If it is assumed that the cell does not respond to wavelengths longer than 3.3 microns, then for 1000 F the percentage-increment scale will show that only about

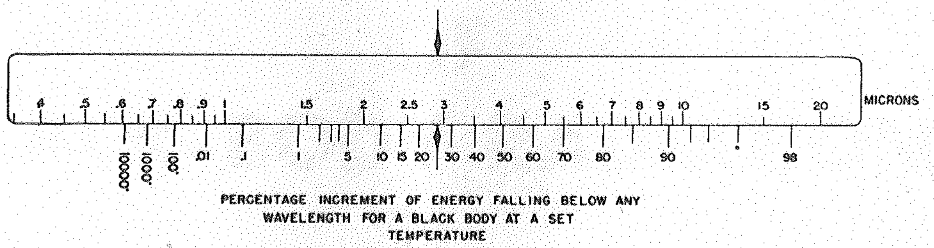

Fig. 5. Illustration of the slide-rule method for computing black-body energy distribution

20 percent of the energy from the source is available to the lead-sulfide cell to be converted into a usable signal. At 1500 F, reading the percentage-increment scale again, 40 per cent of the energy is available for signal at the lead-sulfide cell. At 2000 F, 3.3 microns falls opposite about 57 percent; thus, 57 percent of the energy of the black body at this temperature is available for signal at the lead-sulfide cell.

The complete radiation slide rule has been constructed and made available for distribution in the hope that it will aid in the understanding of the radiation laws and materially simplify calculations connected with problems in illumination, applications of infrared, radiant heating, temperature measurement by radiometry, infrared spectroscopy, spectroradiometry, and heat transfer.

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5-52 (2800)

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Radiation Slide Rule

For The Calculation of Black Body Radiation

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SCHENECTADY, N. Y.

SPECIAL TESTING AND MEASURING INSTRUMENTS FOR . . .

- CHEMICAL ANALYSIS
- COLOR AND OPTICAL EQUIPMENT
- RADIATION DETECTION
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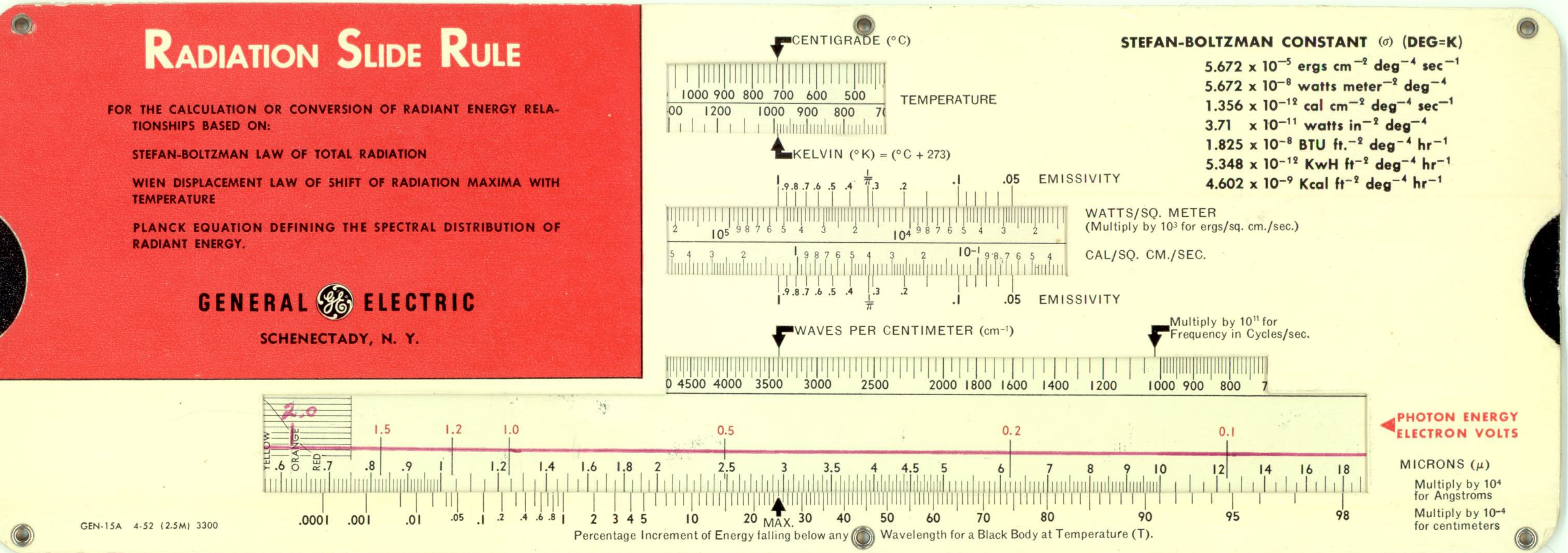
RADIATION SLIDE RULE
FOR THE CALCULATION OR CONVERSION OF RADIANT ENERGY RELATIONSHIPS BASED ON:
STEFAN-BOLTZMAN LAW OF TOTAL RADIATION
WIEN DISPLACEMENT LAW OF SHIFT OF RADIATION MAXIMA WITH TEMPERATURE
PLANCK EQUATION DEFINING THE SPECTRAL DISTRIBUTION OF RADIANT ENERGY.
GENERAL ELECTRIC
SCHENECTADY, N. Y.
CENTIGRADE (°C)
TEMPERATURE
KELVIN (°K) = (°C + 273)
EMISSIVITY
WATTS/SQ. METER
(Multiply by 10³ for ergs/sq. cm./sec.)
CAL/SQ. CM./SEC.
EMISSIVITY
WAVES PER CENTIMETER (cm⁻¹)
Multiply by 10¹¹ for Frequency in Cycles/sec.
STEFAN-BOLTZMAN CONSTANT (σ) (DEG=K)
5.672 x 10⁻⁵ ergs cm⁻² deg⁻⁴ sec⁻¹
5.672 x 10⁻⁸ watts meter⁻² deg⁻⁴
1.356 x 10⁻¹² cal cm⁻² deg⁻⁴ sec⁻¹
3.71 x 10⁻¹¹ watts in⁻² deg⁻⁴
1.825 x 10⁻⁸ BTU ft.⁻² deg⁻⁴ hr⁻¹
5.348 x 10⁻¹² KwH ft⁻² deg⁻⁴ hr⁻¹
4.602 x 10⁻⁹ Kcal ft⁻² deg⁻⁴ hr⁻¹
YELLOW
ORANGE
RED
2.0
PHOTON ENERGY ELECTRON VOLTS
MICRONS (μ)
Multiply by 10⁴ for Angstroms
Multiply by 10⁻⁴ for centimeters
MAX.
Percentage Increment of Energy falling below any Wavelength for a Black Body at Temperature (T).
GEN-15A 4-52 (2.5M) 3300

CHARACTERISTICS OF TUNGSTEN

TRUE TEMP. (°K)	2000	2500	3000	3500
BRIGHTNESS CANDLES/CM²	20 4 6 8 100	2 3 4 5 6 7 8 9 1000	2 3 4 5	
COLOR TEMP. (°K)	2000	2500	3000	3500

FOR TOTAL RADIANT FLUX PER UNIT SOLID ANGLE (STERADIAN) READ ON ENERGY SCALE OPPOSITE $\frac{1}{\pi}$

PERRYGRAF Slide-Chart, Perrygraf Corp., Maywood, Ill.

FAHRENHEIT (°F)

TEMPERATURE

RANKINE (°R) = (°F + 460)

EMISSIVITY

WATTS/SQ. IN.

B.T.U./SQ. FT./HR.

EMISSIVITY

KWH/SQ. FT./HR.

K CAL/SQ. FT./HR.

EMISSIVITY

STEFAN-BOLTZMANN LAW:

$W = \rho\sigma T^4$ or $W = \rho\sigma(T^4 - T_0^4)$

WIEN'S DISPLACEMENT LAW:

$\lambda m = \frac{a}{T}$

PLANCK'S EQUATION:

$W_\lambda = C_1\lambda^{-5}\left(e^{\frac{C_2}{\lambda T}} - 1\right)^{-1}$

W_λ = RADIANT FLUX PER UNIT AREA PER UNIT INCREMENT OF WAVELENGTH.

W = TOTAL RADIANT FLUX EMITTED PER UNIT AREA

T = ABSOLUTE TEMPERATURE OF RADIATING BODY (°K)

T_0 = ABSOLUTE TEMPERATURE OF SURROUNDINGS (°K)

λ = WAVELENGTH IN MICRONS

λm = WAVELENGTH IN MICRONS OF MAXIMA OF BLACK BODY CURVE

a = CONSTANT FOR BLACK BODY = 2897 MICRON DEGREES

$C_1 = 2\pi c^2 h = 3.740 \times 18^8$ } FOR W_λ IN WATTS PER SQ. METER

$C_2 = hc/k = 1.4385 \times 10^4$ } PER MICRON FOR 2π SOLID ANGLE

e = 2.718 NAPERIAN BASE

ρ = EMISSIVITY FACTOR (BLACK BODY = 1)

σ = STEFAN-BOLTZMAN CONSTANT

c = VELOCITY OF LIGHT = 2.99776×10^{10} cm./second

h = PLANCK'S CONSTANT = 6.6234×10^{-27} erg seconds

k = BOLTZMAN'S CONSTANT = 1.38032×10^{-16} erg per degree